CAN LEGAL ACTORS PLAY EQUILIBRIUM STRATEGIES? TWO DUBIOUS ASSUMPTIONS IN THE GAME-THEORETIC ANALYSIS OF THE LAW

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ABSTRACT

Strategic actors often require a great deal of informational and computational resources to calculate game-theoretic equilibria, and scholars need a precise accounting of incentives in order to model the decisions faced by these actors. Rarely are both of these conditions—player rationality and payoff quantifiability—met when scholars attempt to model the behavior of individual actors in the law (especially the behavior of lay people such as jurors, litigants, or criminals). Behavioral economists have proposed posthoc adjustments to account for the failure of player rationality, but these adjustments—if they are indeed correct—make the pursuit of equilibrium impossible for any actor with realistic informational and cognitive resources. This Article discusses the necessary conditions for the emergence of equilibrium strategies, and identifies examples of legal scholarship in which the use of equilibrium solution concepts is problematic because of the improbability that these conditions are met.

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I. INTRODUCTION

Knowledge of game theory does not make any one a better card player, businessman, or military strategist . . .

—Anatol Rapoport (1962)1

Law and economics theory attempts to define legal outcomes as the product of individuals’ rational pursuit of self-interest. In its most doctrinaire form, the economic analysis of law assumes that every legal actor is a homo economicus, strategically optimizing expected utility and believing that others do the same (and hold the same beliefs, and beliefs about beliefs, ad infinitum). This view of decisionmaking is required for the use of Nash equilibrium, which is increasingly prominent in the study of law.

In addition to the assumption of economic rationality, the use of equilibrium solution concepts also assumes that strategic actors (and scholars modeling their decisions) can accurately identify the payoffs derived from each possible outcome for each individual involved. Indeed, one cannot even define a game without identifying these payoffs. Game theorists express these payoffs in the form of utility—a measure that encapsulates the total value that an individual assigns to an outcome, including any value derived from the fate of other players.

Rarely are both of these assumptions—player rationality and payoff quantifiability—justified in the study of law. This Article identifies two major stumbling blocks in the use of equilibrium solution concepts.

II. PLAYER RATIONALITY

The first requirement for a legal actor to play equilibrium strategies is for that actor to be an expected utility maximizer. Unfortunately, most people do not maximize expected utility in the face of risky choices.2

2. See Maurice Allais, Le Comportement de L’Homme Rationnel Devant le Risque: Critique des Postulats et Axiomes de L’Ecole Americaine, 21 ECONOMETRICA 503, 504 (1953) (“Everybody recognizes the fact that man in reality does not behave according to the principle of Bernoulli.”); Daniel Bernoulli, Specimen Theoriae Novae de Mensura Sortis [Exposition of a New Theory on the Measurement of Risk], 5 COMMENTARII ACADEMII SCIENTIARUM IMPERIALIS PETROPOLITANAE 175 (1738) (Russ.), translated in 22 ECONOMETRICA 23, 24 (Louise Sommer trans., 1954) (“[N]o valid measurement of the value of a risk can be obtained without consideration being given to its utility . . . . However, it hardly seems plausible to make any precise generalizations since the utility of an item may change with circumstances.”); Daniel Ellsberg, Risk, Ambiguity, and the Savage Axioms, 75 Q.J. ECON. 643, 667 (1961) (reporting that “our subject does not actually expect the worst, but he chooses to act ‘as though’ the worse were somewhat more likely than his best estimates of likelihood.”)
Although a range of behavioral theories (such as the Cumulative Prospect Theory⁴) have been proposed to correct or account for deviations from expected utility maximization, there are two problems with these corrections: (1) they may be wrong; and (2) they make the calculation of equilibria essentially impossible.

Behavioral alterations to expected utility theory are easy to prove wrong, but difficult to prove right because they are crafted inductively in response to unexpected behavior and cannot be tested on the same types of games that inspired their formulation. That is, an out-of-sample test is required to avoid curve fitting. Moreover, the basis for these alterations is the typically unjustified view that humans engage in something that is, in its essence, expected utility maximization, but deviate from this path because of various cognitive biases. Just as Ptolemaic astronomers used planetary epicycles to defend geocentrism from contradictory empirical evidence, behavioral economists use adjustments for cognitive bias to defend Cartesian rationality from contradictory empirical evidence. Gigerenzer argues that explaining human rationality as an approximation of mathematical logic ignores the fact that human reasoning is a product of evolution.⁴ Instead, he and his colleagues view rationality as growing out of evolutionarily adaptive heuristics and cite the classic paradoxes of human behavior as evidence for these heuristics:

The priority heuristic predicts (a) Allais paradox, (b) risk aversion for gains if probabilities are high, (c) risk seeking for gains if probabilities are low (e.g., lottery tickets), (d) risk aversion for losses if probabilities are low (e.g., buying insurance), (e) risk seeking for losses if probabilities are high, (f) the certainty effect, (g) the possibility effect, and (h) intransitivities.⁵

Even if the expected utility hypothesis is fundamentally correct, and deviations can be explained by a small number of cognitive biases, equilibria will be essentially impossible to calculate for any observer endowed with less than omniscient intelligence. Consider the Cumulative

would indicate, In either case, he violates the Savage axioms . . . ”); Daniel Kahneman & Amos Tversky, Prospect Theory: An Analysis of Decision Under Risk, 47 ECONOMETRICA 263, 263 (1979) (“Choices among risky prospects exhibit several pervasive effects that are inconsistent with the basic tenets of utility theory.”).

3. See Kahneman & Tversky, supra note 2, at 263–64 (introducing prospect theory as an alternative model of decisionmaking in which value is assigned to gains and losses, rather than to final assets, and noting “a prospect is acceptable if the utility resulting from integrating the prospect with one’s assets exceeds the utility of those assets alone”).


5. Id. at 409.
Prospect Theory, which involves a nonlinear function with five adjustable parameters. Is this function common knowledge? Is each player’s level of risk aversion and sensitivity to reference point framing effects common knowledge? Do players calculate equilibria based on a modified game that takes into account the alterations attributable to the Cumulative Prospect Theory? This seems impossible.

Assume for the time being that jurors, judges, legislators, criminals, litigants, and the like are all expected utility maximizers. These individuals still require a great deal more informational and computational resources to calculate strategic equilibria than they do merely to best respond to another’s action. Informationally, actors must know exactly the actions, information, and strategies available to all players in the game, and the payoffs for all outcomes, in order to calculate equilibrium. Computationally, mixed-strategy Nash equilibria are very difficult to calculate for nonzero sum games, falling into the “Polynomial Parity Arguments on Directed Complexity” class. The computational time required to solve such tasks grows exponentially with the complexity of the inputs, so finding the equilibria of even mildly complex two-player games may be computationally intractable. Empirically, human subjects are often incapable of finding even the simplest equilibrium solutions.

An example of the computational complexity of calculating equilibrium strategies can be found in Leshem’s excellent work on the

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6. See Kahneman & Tversky, supra note 2, at 274–84.
10. Id. at 516 (describing problems faced by Nash equilibrium theory and noting that “a bimatrix game may have no equilibrium”).
benefits of the right to silence for innocent defendants.12 The model in Leshem’s article captures the essentials of choosing whether to invoke one’s Fifth Amendment right to silence, and as a work of formal theory the solution is unimpeachable. But do defendants and jurors—untrained in formal logic and probability theory, let alone game theory—really calculate equilibria to this game? As a rough measure of the complexity of such a task, Leshem offers a detailed technical appendix to explain how in equilibrium the right to silence benefits innocent defendants.13

Empirically, even trained economists are generally not “economically rational” in the sense of best responding to equilibrium beliefs. In an experiment involving the Keynesian Beauty Contest (a guessing game requiring mutual best response),14 Bosch-Doménech found that 81 percent of economists at a game-theory conference did not select the Nash equilibrium.15 Keynes famously argued that when players attempt to best respond to each other, as investors do on the stock market, “[w]e have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practice the fourth, fifth and higher degrees.”16 Clearly, Keynes does not expect most investors to iterate their beliefs infinitely, as an actor would in equilibrium.

Game theorists frequently counter the empirical evidence against equilibrium with the criticism that laboratory subjects are untrained individuals playing games for the first time.17 Over time, they argue, subjects would learn to play equilibrium strategies and hold equilibrium beliefs.18 From a theoretical perspective, however, the prominent models of learning do not necessarily predict convergence on equilibrium.19

13. Id. at 413–15.
17. See, e.g., Chou et al., supra note 11, at 160 (“[T]he absence of appropriate strategic behavior is not due to an inability to think or act strategically” but rather because subjects “are substantially unaware of important elements of the experimental environment.”).
18. See, e.g., id. at 178 (“We know that with considerable repeated play this game, like others, tends to converge to the equilibrium.”).
Empirical tests of learning in laboratory experiments also show that behavior does not necessarily converge to Nash equilibrium, especially in positive-sum, extensive-form and nondominance solvable games. And even if learning does lead players toward an equilibrium outcome, when there are multiple equilibria (as is the case in most real-world social processes), nonequilibrium strategies in initial rounds often determine which equilibrium players converge on.

III. PAYOFF QUANTIFIABILITY

A fundamental premise of game theory is that, when individuals interact, we can identify the payoffs derived from each possible outcome for each individual involved. Indeed, we cannot define a game without identifying these payoffs. Game theorists express these payoffs in the form of utility—a measure that encapsulates the total value that an individual assigns to an outcome, including any value derived from the fate of other

results leave open the possibility that even where there is a unique strict Nash equilibrium, play does not always converge to it.”); Drew Fudenberg & David M. Kreps, Learning Mixed Equilibria, 5 GAMES & ECON. BEHAV. 320, 321, 338–47 (1993) (examining mixed-strategy games where convergence on equilibrium does not occur). For an excellent review, see Drew Fudenberg & David K. Levine, Learning and Equilibrium, 1 ANN. REV. ECON. 385, 387 (2009) (“[P]lay only converges to Nash equilibrium in some classes of games, and when play does not converge, the environment is not stationary and the players’ rules may perform poorly.”).


21. See Emerson M.S. Niou & Peter C. Ordesbook, “Less Filling, Tastes Great”: The Realist—Neoliberal Debate, 46 WORLD POL. 209, 210 (1994) (“Because virtually every ongoing social process possesses a multiplicity of equilibria, opportunities to cooperate and the concomitant problem of coordinating to one of these equilibria are omnipresent.”).

22. John B. Van Huyck, Joseph P. Cook & Raymond C. Battalio, Adaptive Behavior and Coordination Failure, 32 J. ECON. BEHAV. & ORG. 483, 484 (1997) (“The selected equilibrium is path-dependent, that is, the equilibrium predicted to emerge depends on the historical accident of the initial condition, rather than on deductive concepts of efficiency.”).
players.

A. OBJECTIVE PAYOFFS

In order to use objective, measurable rewards as payoffs (for example, “player one receives one dollar for outcome X”), one must assume that the measured payoffs are not confounded by other considerations. Two obvious potential confounds in a game that involves players earning measurable rewards such as these are the shadow of future interactions\(^\text{23}\) and the presence of side payments,\(^\text{24}\) but a properly modeled game would account for these confounds. More subtle and endemic, and harder to account for, are deviations from a simple one-to-one relationship between the quantity of measurable rewards earned by an individual and the utility that the individual derives from the rewards. Asserting such a one-to-one relationship requires two particularly bold assumptions: (1) each player’s utility increases linearly with measured earnings; and (2) each player understands the earnings of all others well enough to use those earnings to predict their actions, and yet does not care how much the others earn.

Unfortunately for those using objectively quantifiable payoffs such as dollars (as do nearly all experimental economists), most people derive decreasing returns from additional income.\(^\text{25}\) Relaxing the linearity assumption, however, imposes immense informational and cognitive burdens on the players. If players’ utilities are allowed to deviate from a linear function of dollars, then the functional form of all players’ dollar-derived utility must be common knowledge. If you and I play a game, and your utility is a positive but decreasing function of income, calculating a mixed-strategy equilibrium would require me to know the form of your utility function and for you to know the form of mine, and for both of us to know that we know each other’s functions, and know that we know them, to an infinite regression.

Also unfortunate for those using objectively quantifiable payoffs is the fact that human beings usually care about what happens to others. This trait

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may be passed down socially,26 and perhaps genetically.27 We even care about humans whom we have never met, nor will ever meet.28 If any player cares about the utility of others, we run into a very thorny problem: all players must understand the value the others put on their opponents’ earnings, and again, this must be common knowledge. This seems to the author an unrealistically heavy informational burden for players to bear.

B. SUBJECTIVE PAYOFFS

When the assumption of a linear relationship between objective rewards and utility seems unreasonable, or when players face no objective rewards, modelers may attempt to identify subjective payoffs. First we will consider subjective payoffs as a function of objective rewards; then we will investigate subjective payoffs that are more psychological in nature, such as those used in models of judicial behavior.

The first of the two assumptions listed in the discussion of objective payoffs above—a linear relationship between objective rewards and utility—may be relaxed with the adoption of a utility function that increases with earnings but at a decreasing rate. Graetz and his colleagues did this in their seminal work on tax evasion, which identified the Nash equilibrium of a game between taxpayers and the IRS, with taxpayers gaining utility from income at a decreasing rate.29 Unfortunately, this introduces new complications: How does the IRS know the function whereby taxpayers convert dollars into utility? Does the taxpayer know that the IRS knows this function? And why should we believe that all taxpayers use the same formula? This, of course, is to say nothing of the cognitive demands on taxpayers of calculating equilibrium.

To relax the assumption that players are unconcerned with each other’s utilities, psychologists and behavioral economists sometimes


27. See David Cesarini et al., Heritability of Cooperative Behavior in the Trust Game, 105 PROC. NAT’L ACAD. SCI. U.S. 3721, 3724–25 (2008) (explaining evidence which suggests that genetics may play a role in shaping a person’s trust and trustworthiness).

28. See Ernst Fehr & Urs Fischbacher, The Nature of Human Altruism, 425 NATURE 785, 785–87 (2003) (finding that in some instances humans will act altruistically regardless of previous or future interactions).

advocate the use of “value orientations.” A value orientation includes assigning weights on the earnings of oneself and others and allowing for negative weights on the earnings of others. The use of value orientations, however, is problematic. It introduces an atheoretical element into utility functions—a free parameter that can be used as a posthoc justification for experimental data that violates rationality. But even if individuals’ orientations are estimated using data from one set of games and confirmed using data from a second (out-of-sample) set of games, their distortion of utility functions prevents the calculation of equilibrium in much the same way that decreasing returns to income does: players cannot calculate equilibria unless they have a method for eliciting each other’s value orientations and establishing that those value orientations are public knowledge.

This is an especially thorny problem because players may have incentives to misrepresent their value orientations to obtain a more desirable equilibrium outcome. Indeed, David Levine argues that the value a subject derives from his opponent’s income (call this his altruism level) actually depends on what he believes his opponent’s altruism level to be.

The table below helps illustrate value orientations and the weight the individuals assign depending on their orientation.

<table>
<thead>
<tr>
<th>Orientation</th>
<th>Maximand</th>
<th>Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altruistic</td>
<td>Other’s welfare</td>
<td>Me = 0; You = 1</td>
</tr>
<tr>
<td>Cooperative</td>
<td>Social welfare</td>
<td>Me &gt; 0; You &gt; 0</td>
</tr>
<tr>
<td>Individualistic</td>
<td>Own welfare</td>
<td>Me = 1; You = 0</td>
</tr>
<tr>
<td>Competitive</td>
<td>Own welfare relative to other’s</td>
<td>Me = 1; You = -1</td>
</tr>
<tr>
<td>Aggressive</td>
<td>Inverse of other’s welfare</td>
<td>Me = 0; You = -1</td>
</tr>
</tbody>
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One problem with assigning subjects social-preference labels based on their behavior is that there is a multiplicity of plausible psychological motivations for any given action in the lab. Consider an ultimatum game in which the first player proposes how to split ten dollars between himself and
a second player, and the second player can either accept the split or insist that neither player earn any money. When the second player accepts a low offer, is he being generous? Opportunistic? Self-interested? Socially regarded? Is he consenting to an unfair deal because he believes that inequality is natural and acceptable? Or is he cooperating despite deep resentment over unfair treatment? These are all interesting questions, but economic games are likely not the best tool for unraveling such mysteries.

When it comes to purely psychological utilities—or payoffs that are not simply a function of some quantifiable reward—we encounter serious measurement problems. For example, judges—especially those in an independent judiciary—often have no formal incentives when deciding cases, so their incentives are often intrinsic, and intrinsic motivations simply cannot be measured. An economically rational and purely self-interested judge who has a life-tenure position and no further ambitions may not care at all about the outcome of any case he could legally hear, since he is barred from adjudicating cases in which he has a personal interest. Perhaps a homo economicus in the judiciary, absent further career ambitions, would be satisfied to flip coins and enjoy his ample leisure time. There may be observable predictors of a judge’s “type,” or level of adherence to judicial standards: his record of decisions and his career trajectory may be informative. But any game-theoretic model involving a judge without formal incentives should address how other players go about estimating the judge’s payoffs.


34. As Drahozal puts it, U.S. federal judges “are appointed by the President and confirmed by the Senate, with life tenure and a salary secure from legislative reduction. Federal judges cannot rule in cases in which they have a direct financial interest, and their income is essentially unaffected by the number of cases they decide or the quality of their decisions.” Drahozal, supra note 33, at 473 (footnotes omitted).

35. See Drahozal, supra note 33, at 474–77.